Evaluating Cash Benefits as Real Options for a Commodity Producer in an Emerging Market

Fernando Antonio Lucena Aiube
Petrobras and PUC-RJ
Edison Americo Huarsaya Tito
Petrobras and PUC-RJ

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Abstract

The amount of cash a firm should maintain is an old problem tackled by finance literature. The recent advances in finance, mainly in the derivatives area, has opened the opportunity to revisit this subject. Cossin and Hricko (2004) studied the benefits of cash holdings using the Real Options approach. We follow their ideas extending the problem to a specific commodity producer firm. We evaluated the benefits considering that raising capital takes time and also the benefit of the avoiding the issue of securities at unfavorable moment. We used numerical procedures to solve the problem. Despite the fact that the results are not totally intuitive, we verified that the timing benefit is much more relevant than that of avoiding underpricing benefit and that firms in emerging economies have greater advantage holding cash than those in developed economies.

Keywords: Real Options, Cash holdings, Commodity producer.

1 Introduction

Consider the problem of a firm that needs to invest in a project. It has the option to increase its cash to provide the funds or raise money outside. What is the best choice? Better, what is the optimal amount of cash a firm should have? Cash holdings has a cost related to the opportunity cost of having to invest in liquid assets for short periods of time with low return. Such cash should be invested in the project if the sources of capital were at hand at low cost. In general, the amount needed to invest is not available and it takes time to raise money in the markets. Although this is a well known problem, the finance literature has not devoted special attention to it. Keynes (1936) addressed the reasons a firm would maintain cash in twofold: (i) transaction cost - the firm should have cash to avoid selling lesser liquid assets at lower price; (ii) precautionary - the firm should have liquid assets to avoid raising capital at unfavorable moment. Myers and Majluf (1984) identified inefficiencies in raising funds by firms when their securities are undervalued by the markets. Their approach was based on game theory. Cossin and Hricko (2004) used the advances of finance literature claiming that the subject needs further development. The authors used derivative pricing to evaluate the benefits of cash holdings. They divided the problem in valuating two different options: (i) timing option - the firm needs a certain amount of capital to invest in real projects but it is not at hand and takes time to raise this capital in the market; (ii) underpricing option - raising capital not only takes time but also can be done when the firm is undervalued. Evaluating these options the authors measured the benefits of holding cash. At the end they combined both options to solve the entire problem. Some unexpected results were shown. In this article we follow the same approach of the last study, extending the problem to a specific case where the firm is a commodity producer and is located in a emerging market. These extensions are based on different assumptions for the stochastic processes followed by value of the project and level of security undervaluation.

The paper is organized as follows: section 2 details the Cossin and Hricko's model, section 3 deals with the extended model to evaluate the cash benefit for a commodity producer firm in an emerging market, section 4 concludes the work.

2 The Cossin and Hricko's model

Consider a firm that has an opportunity of investing a fixed amount in a real project. At time zero it has to choose the source of capital. It can increase its cash or raise the funds in the capital market. If the funds come from the market, the firm could use equity or debt issues. In both cases it takes time to have the capital ready to invest in the project. The authors assumed that the funds would be ready at time T. Relying on outside financing could be suboptimal since the optimal time for investment should be before T.

The authors divided the benefits of having cash into two different forms. First they evaluated the option of having funds and investing in the project before the time required to issue securities. Second, at time T, when issuing securities, can be a moment that the firm is undervalued (for any reason its stocks are being negotiated at lower value than managers believe would be the fair price). They evaluated the option that having cash avoids the issuance of under priced securities. In this situation the project can only be undertaken at time T. The complete problem is the combination of these two options: the firm can raise the cash today and invest it in any moment until T.

2.1 The timing option

The timing option is the benefit of having cash and investing in the project before the time required in case of issuing securities. The authors started with the model first developed by McDonald and Siegel (1986) in which the value V of the project evolves like a geometric Brownian motion (GBM)[1]

$$\frac{dV_t}{V_t} = \alpha dt + \sigma_V dW_{V_t} \tag{1}$$

where α is the drift of the process, σ_V is volatility of the return of the project and dW_{V_t} is the increment of the standard Brownian motion. Consider K is the fixed investment to undertake the project. And the NPV of the project today is $V - K$. Also consider that d is the loss for waiting to undertake the project. This value represents what the firm will loose if competitors undertake the project before. Or also, it can be interpreted as a decreasing value of the project since its costs can rise. Thus d is a percentage loss per period. Similar to the derivative pricing in finance literature, d is the analogous of the dividend yield of a stock [2].

If the firm decides to finance the project with cash, it can take part of this cash, K, and invest in the project to get the value V. And this can be done at any time until T. This is equivalent to an american option, more specifically to an american call option in which the investor exercise his or her rights buying the stock at K (the exercise price) any time before the maturity. On the other hand, if the firm decides to raise external capital that will be available only at T, it has a european call option (the option

[1] Antecedents of the McDonald and Siegel (1986) include Myers (1977) and Tourinho (1979) as pointed out by Dixit and Pindyck (1994).

[2] In the risk neutral measure this process is written as $\frac{dV_t}{V_t} = (r - d)\, dt + \sigma d\tilde{W}_t$

to undertake the project can only be exercised at time T when the funds are available). The authors assumed that the firm does not have interest postponing after T. Thus at the time when the decision is made (finance the investment with cash or raise external funds), the benefit of having cash is the difference between an american and a european call options:

$$B_t = C_t(T) - c_t(T) \tag{2}$$

where B_t represents the timing benefit measured at t, $C_t(T)$ is the american call maturating at T and evaluated at t and $c_t(T)$ is the european one. This last option is evaluated by the standard Black and Scholes formula. To calculate the american option the authors used the Barone-Adesi and Whaley (1987) approximation. We used the least square Monte-Carlo (LSM), presented at Longstaff and Schwartz (2001), to calculate the american option at time $t = 0$. The results we found are quite similar to the authors. Figure 1 shows the value of the benefit with the loss rate for different volatilities (or uncertainties across the value of the project). In our calculation we used the same values for the variables the authors used, which are $V_0 = 100$, $K = 90$, $r = 0.08$ and $T = 0.25$. Here V_0 is value of the project at time zero and r represents the risk free interest rate. It was assumed that the time required to raise external funds is 3 months. To proceed with the calculation using LSM we generated the paths in the risk neutral measure as usual in derivative pricing theory. As can be observed from Figure 1 the benefit of having cash increases as the loss of waiting increases, this is a natural result. The other conclusion we reached is related to the fact that as volatility increases the benefit decreases. It does not seems intuitive since for riskier projects firms will have less benefit having cash. This counterintuitive result comes from the fact that the model considers the investment as deterministic. Indeed if it were modeled as stochastic the result would be different.

2.2 The underpricing avoidance benefit

The underpricing avoidance benefit comes from the fact that the firm can issue securities at T when its shares are undervalued by the market for some reason. The authors consider that the amount of undervaluation X, is given by a mean-reverting process

$$dX_t = a(b - X_t)\,dt + \sigma_X dW_{X_t} \tag{3}$$

where a is the mean reverting speed, b is the long run amount of undervaluation, σ_X is the volatility of undervaluation and dW_{X_t} is the standard

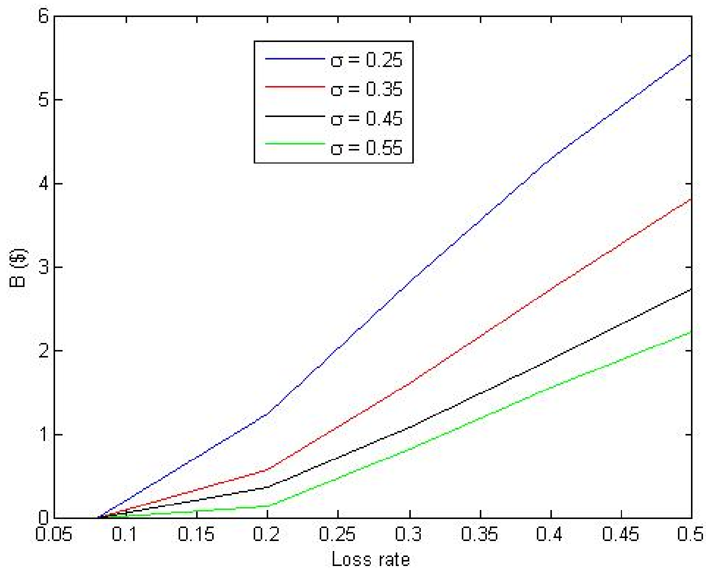

Figure 1: Timing benefit

Brownian process. Also the processes given in equation (1) and in equation (2) are correlated, then $\rho dt = dW_{V_t} dW_{X_t}$. If there is no undervaluation, raising money at time T and investing in the project means that the firm has a european option. On the other hand if at time T the firm's securities are undervalued, raising the capital and investing in project means the firm has a european option with payoff given by $max\,(V_T - K - X_T)$. This is an option with stochastic payoff. Indeed the firm will have a decrease in the project value by the amount of X. It can be expressed as

$$B_t = c_t\,(T) - c_t^{under}\,(T) \tag{4}$$

where $c_t^{under}\,(T)$ stands for the european option maturing at time T and evaluated at t under the condition of underpricing. We proceed as above implementing numerical calculations with simulations taken under the risk neutral measure. Again, the results (based on the same parameters) are similar to those found by the authors. It is worth noting that here the benefit decreases as the loss rate of waiting increases, not a very clear result. Also the result implies that the underpricing avoidance benefit is smaller than the timing benefit.

2.3 The timing and underpricing avoidance benefits

The combination of both risks can be interpreted as: holding cash allows the firm to undertake the project any time before T and eliminates the problem of raising money at unfavorable moments. The first one is an american option maturing at T an the second is a european option with underpricing risk and the same maturity. Then it can be written that

$$B_t = C_t(T) - c_t^{under}(T) \tag{5}$$

We used numerical procedures and found that the benefit decreases with volatility, a result of the investment model used as explained above. Also the benefit increases with the loss rate for waiting, d. It behaves as if timing benefit dominates the underpricing counterpart.

3 The extended model

In this section we extend the previous model replacing the stochastic processes used earlier to focus on a commodity producer in an emerging economy. We are going to do this in twofolds: first the project value V for a commodity will be the mean-reverting; and second, the process for the undervalue amount X will have a jump component.

3.1 The timing option

The use of GBM covers the classical problems in Real Options. If we think a more specific model for a commodity we need another approach to describe the evolution of the value of the project. Dixit and Pindyck (1994) points the mean-reverting process as an alternative to the GBM. The value of the project is then expressed by $dV_t = \eta\left(\bar{V} - V_t\right) V_t dt + \sigma V_t dW_t$. This type of evolution is appropriate for situations where the variable evolves stochastically around the long run average $\bar{V}$. It is a well known fact in finance that the commodity prices typically follows this type of process. If the value of a project is roughly linked to the process product price, the process for V would be a mean-reverting for a commodity case. We adopted the mean-reverting process to represent the value of a project of the commodity industry. Consider that V is given by

$$\frac{dV_t}{V_t} = k(\theta - \ln V_t)\, dt + \sigma_V dW_{V_t} \tag{6}$$

where k is the speed of reversion, θ is defined bellow, σ_V is the volatility of V and dW_{V_t} is the standard Brownian process. Writing $Z_t = \ln V_t$ and using

Ito's lemma we reach

$$dZ_t = k\left(\alpha - Z_t\right)dt + \sigma_V dW_{V_t} \tag{7}$$

where α is the log of long run project value and is given by $\theta - \frac{\sigma^2}{2k}$. Now we are going to evaluate the benefit related to timing option as earlier. The equation for the value of the benefit of having cash and investing in the project at any time before maturity is given by equation (2). We are facing the problem of calculating the american and european options for the process considered in equation (6). We used the LSM algorithm to calculate the american option and the simple Monte-Carlo simulation for the case of the european option. All the paths were generated in the equivalent martingale measure. Figure 2 is the result of the simulation. The benefit increases as the volatility σ_V

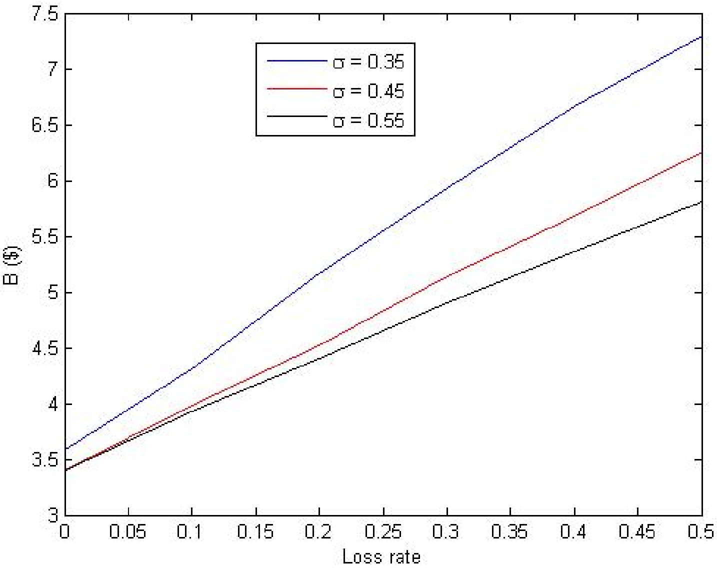

Figure 2: Timing benefit

decreases and as the loss rate for waiting increases. We used the same values as before, i.e. $V_0 = 100$, $K = 90$, $r = 0.08$, $T = 0.25$, $k = 2$, $\lambda_V = 0.02$ and $\theta = 4.45$. Here λ_V represents the market price of risk for the project, V. One can observe that for a loss rate of 25% and volatility of 35% the benefit is approximately \$5.5, this is a significant value compared, for example, to the NPV of the project, of \$10 if immeadeatly undertaken. We made sensitivity with the speed of mean-reversion k and the result follows in subsection 3.3.

3.2 The underpricing avoidance benefit

If the firm decides to issue securities we have seen that there is a possibility of doing it when its shares are undervalued. Despite this fact, raising funds to invest in the project can be worthwhile. It depends on how robust the project is. Nonetheless, there is an advantage of having cash and avoiding raising capital. The previous modeling established that the amount of undervaluation X follows a mean-reverting process. Now we are going to consider the case where the firm is in an emerging economy. In this situation, the firm is inside a more volatile environment. For example, suppose an outside crisis in equity markets, there is immediately a contagion in the country where the firm is based. And in general this effect is amplified. Another situation is an internal crisis, in this case the capital outflows from the emerging economy. All this flow of capital is common in a global economy making the emerging markets more volatile than developed economies. Whatever the reason, the undervaluation process needs to capture this effect. For this purpose we introduced the jump component in equation (3) to describe the X evolution. Now the model is written

$$dX_t = a\left(b - X_t\right)dt + \sigma_X dW_{X_t} + Y dq_t \tag{8}$$

where Y is the magnitude of the jump and is given by $Y \sim N\left(\mu_J, \sigma_J^2\right)$, μ_J and σ_J^2 are the mean and variance of jump size, respectively. dq_t represents the Poisson process with intensity parameter λ_J. All others parameters in equation (8) were previously defined. Also there is no correlation between the Poisson process with the diffusion process of X nor with the diffusion of V. The correlation between V and X is given by $\rho dt = dW_{V_t} dW_{X_t}$.

The benefit of underpricing avoidance is given by equation (4). We need to calculate two european options: the first is based on equation (6) where the payoff is given by $max\left(V_T - K\right)$ and the second is the option with stochastic payoff based on equations (6) and (8) and given by $max\left(V_T - K - X_T\right)$. Both options were calculated using Monte-Carlo simulation considering the risk neutral measure. Figure 3 presents the results. The parameters in this simulation, used in process V, are: $V_0 = 100$, $K = 90$, $r = 0.08$, $T = 0.25$, $k = 2$, $\lambda_V = 0.02$ and $\theta = 4.45$. On the X process we used: $a = 0.2$, $b = 3$, $X_0 = 0$, $\sigma_X = 0.5$, $\mu_J = 0.9$, $\sigma_J = 0.6$ and $\lambda_J = 0.5$. The correlation between both is $\rho = -0.2$. We can observe that the the benefit of avoidance issue of securities is much smaller than that resulting from the timing benefit. It is obvious that as the mean of jump size μ_J increases the benefit also increases. And the same is true for the intensity of jumps λ_J, we confirmed these results. We observe the same behavior of the previous model in which the

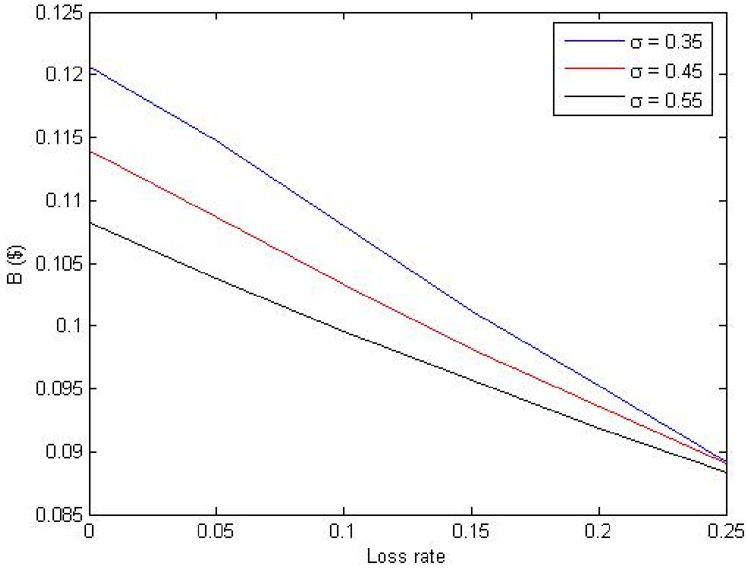

Figure 3: Underpricing avoidance benefit

benefit decreases as the volatility σ_V increases and the same for the loss rate for waiting. Comparing the processes of X with jumps and without jumps we conclude that the inclusion of jumps makes option $c_t^{under}(T)$ value less and hence increase the benefit. So, having cash to avoid undervaluation risk seems more interesting in emerging economies.

3.3 The timing and underpricing avoidance benefits

The next step is to combine both benefits as done earlier. So the equation of the total benefit is given in equation (5). We have to evaluate the american option based on the dynamics given in equation (6) and the european option under the risk of underpricing which involves computing the dynamics given in equations (6) and (8). We proceeded with calculations as before and Figure 4 shows the results. As expected, given the magnitude of these two types of benefits, the result is almost the same as that presented in Figure 2. The benefit decreases with the volatility and increases with loss rate for waiting to invest. We proceeded with sensitivity analysis for parameter k (speed of reversion) in process V . We observed that as k increases the overall benefit decreases.

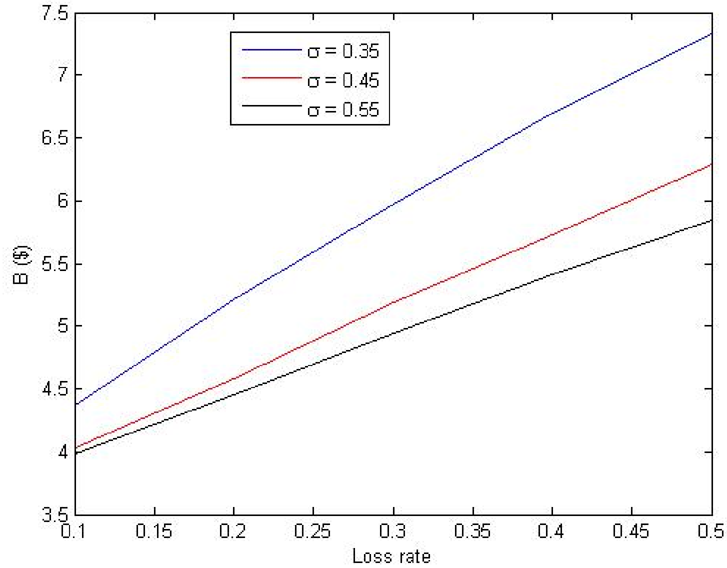

Figure 4: The timing and underpricing avoidance benefits

3.4 Portfolio analysis

We are going to analyse a situation where a firm does not have only one project, but a portfolio. For practical reasons let us consider two different projects. Project one has value V_1 and its dynamics is given by

$$\frac{dV_{1_t}}{V_{1_t}} = k_1 \left(\theta_1 - \ln V_{1_t}\right) dt + \sigma_{V_1} dW_{V_{1_t}} \tag{9}$$

Project two has a similar dynamic given by

$$\frac{dV_{2_t}}{V_{2_t}} = k_2 \left(\theta_2 - \ln V_{2_t}\right) dt + \sigma_{V_2} dW_{V_{2_t}} \tag{10}$$

Both projects are correlated through to X: $\rho_1 dt = dW_{V_{1_t}} dW_{X_t}$ and $\rho_2 dt = dW_{V_{2_t}} dW_{X_t}$. There is no correlation between the diffusion in equation (9) and the jump process in equation (8). The same is valid for equation (10). It is worth noting that, although correlate through Brownians, these two projects are physically independent. This means that they can be undertaken simultaneously, i.e. labor and equipment are not shared between them. In Figure 5 we have the result of the overall benefit, which is the timing and underpricing avoidance benefits for the portfolio. The calculations were done as before and the parameters for both projects are: $V_{1_0} = 100$, $K_1 = 90$, $k_1 = 2$, $\lambda_{V_1} = 0.02$, $\theta_1 = 4.45$ and $V_{2_0} = 80$, $K_2 = 60$, $k_2 = 2$, $\lambda_{V_2} = 0.03$ and

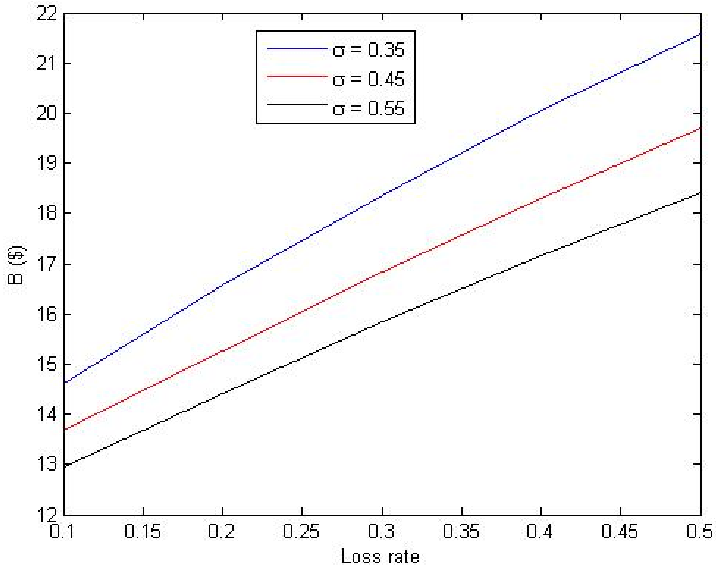

Figure 5: Overall benefit for the portfolio

$\theta_2 = 4.09$. If the portfolio was undertaken now, the NPV is \$30. Considering the volatility in both projects 35% and a rate loss of waiting equal to 30% we get a total benefit of \$18 approximately, which is a significant amount compared to the project NPV.

4 Conclusion

In this paper we evaluated the benefits of holding cash for a commodity producer firm in an emerging economy through Real Options analysis. We extended the Cossin and Hricko's model incorporating both conditions. First, the commodity producer firm has the dynamics of the project value represented by a mean-reverting process, likewise the behavior of commodity price is a well known fact in finance literature. On the other hand the risk of raising funds when the firm's securities are undervalued is increased in an emerging economy. For this reason we added a jump process in the amount that represents the undervaluation behavior. In all these framework we observed that the benefit is greater when the uncertainty in economy is lesser. This is not a very intuitive result but it is related to the model we used to describe the investment. We have used a fixed investment. The parameters used in jump process, that amplify the undervaluation, behaved as expected: (i) the higher the jump size mean the higher is the benefit, (ii) the same for the jump intensity. Also, the firm within the emerging economy should have greater benefit of holding cash compared to that one in a developed economy. Finally, we analysed a more practical issue that is the benefit a firm has financing a portfolio of projects with cash. One can observe that the timing benefit is much higher than the underpricing avoidance benefit. The model can be improved by considering the optimal time to invest. This is an important task in the Real Options framework that was not taken into account. Also, modeling the investment as stochastic would lead to more intuitive results.

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